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The Manual for SEADOS.PAS

A Program for Seasonal Adjustment

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Abstract This manual should serve as a ready reference to assist in operating the *SEADOS* program, which provides some enhanced methods for the seasonal adjustment of economic data. The manual lists the menus of the program, and it gives a thematic account of the facilities of the program. Embedded in the program are brief descriptions of its facilities and its functions. These should also provide guidance for operating the program. The table of contents of this manual contains hypertext links to the sections and subsections.

Keywords seasonal adjustment · frequency response · periodogram analysis

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1 INTRODUCTION

Seasonal adjustment is a process of smoothing a data sequence to remove cyclical fluctuations that are tied to the annual calendar. Such fluctuations are liable to be attributed to an additive seasonal component, which is assumed to be statistically independent of the other motions affecting the data. These may include an underlying trend and a so-called secular cycle or business cycle, which combine to form a trend-cycle function There will also be a component of random noise within the data.

There will be $s = 4, 12$ observations per annum, which is to say that the data are observed either on a quarterly basis or on a monthly basis. The fundamental frequency, which corresponds to one cycle per annum, is therefore $\omega_1 = 2\pi/s$ radians per sampling interval. The harmonic frequencies, which are integer multiples of the fundamental frequency, are limited by the so-called Nyquist frequency of π, which represents the highest frequency that is observable in sampled data.

Taken together, the frequencies $\omega_j = 2\pi j/s; j := 1, 2, \ldots, s/2$, which are the fundamental frequency and its harmonics, are described as the seasonal frequencies. In the case of quarterly data, they are just $\pi/2$ or 90° and π or 180°, whereas, in the case of monthly data, they are $\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, π, or 30°, 60°, 90°, 120°, 150°, 180°.

A perpetually repeating ideosyncratic pattern of fluctuations can be formed as a combination of sinusoidal elements at the seasonal frequencies, with various amplitudes and phase displacements. An evolving pattern of seasonal fluctuations will comprise elements at the adjacent frequencies, which will interact with those at the seasonal frequencies in a manner that varies over time.

1.1 The Nature of the Filters

The *SEADOS* program offers some flexible means of effecting a seasonal adjustment. It removes the seasonal component from the data by applying filters in two directions, moving both forwards and backwards through the sequence of data points. The purpose of filtering in both directions is to avoid displacing the filtered sequence in time—which would induce a so-called phase displacement.

If the seasonal fluctuations are of a regular nature, then the basic filter of the program, which is targeted at the seasonal frequencies, may serve to remove them from the data. In cases where the seasonal fluctuations are less regular, a compound double or triple filter may be called for. The double filter applies in series two offset versions of the basic filter that are targeted at frequencies that lie above and below the seasonal frequencies. The triple filter, which combines the offset filters with the basic filter, provides a more flexible means of seasonal adjustment.

1.2 **The Frequency Response of a Filter**

The effect of a filter on the data can be represented by its frequency response function, which shows how the filter alters the amplitudes of the sinusoidal elements of which the data are composed. The frequencies of these elements range from zero to the Nyquist value.

The frequency response of the basic seasonal-adjustment filter has clefts or notches that descend to zero at the seasonal frequencies, thereby nullifying the corresponding elements. The elements at the adjacent frequencies will be attenuated to an extent that diminishes rapidly as their distances from the seasonal frequencies increases. Elements at frequencies that are remote from the seasonal frequencies should be preserved; and the corresponding values of the frequency response function should be close to unity.

1.3 **The Parameters of the Basic Filter**

The frequency response of the filter should be matched to the periodogram of the data sequence, which shows the amplitudes of the sinusoidal elements of which it is composed. The basic filter has two parameters, which relate to an heuristic model of the processes that have generated the data. The parameter values must be specified by the user of the program, whereas, in the conventional model-based methods of seasonal adjustment, they are liable to be estimated from the data.

The first of these parameters is the pole parameter ρ, which is the modulus of the zeros of the transfer function of the model that maps from a white noise process to the seasonal component. This takes a value in the interval $(0, 1)$. The closer is ρ to unity, the narrower are the clefts in the frequency response function of the de-seasonalising filter.

The second parameter is the signal–noise ratio λ, which is the ratio of the variance of the white-noise process driving the seasonal fluctuations to the variance of the noise component. Its value can remain fixed, while that of ρ can be varied in pursuit of a desirable profile for the frequency response function. However, to achieve an adequate frequency response that will remove all of the seasonal elements, it may be necessary to have recourse to a compound double or triple filter.

1.4 **Removing the Trend from the Data**

If the data have a trend, then the periodogram is liable to be dominated by the elements in the vicinity of the zero frequency. This will make it difficult to assess the contribution of the seasonal component. The problem can be overcome by removing the trend. This can be achieved in the *SEADOS* program by interpolating a polynomial function by a least-squares regression.

An alternative recourse it to employ a low pass Butterworth filter to remove both the trend and the cyclical component from the data and to create a combined trend-cycle function. The periodogram of the residual deviations of the data from the trend or from the trend-cycle function provides the principal diagnostic tool of the program. It shows clearly the range of the frequencies that are contributing to the seasonal fluctuations.

After removing a polynomial trend from the data, the attempt can be made to remove the seasonal component by applying the basic seasonal adjustment filter. Then, the seasonal component can be subtracted from the data sequence to create a seasonally adjusted version. The result will be twofold decomposition of the data comprising the seasonal component and the seasonally-adjusted data sequence, which is a combination of the trend, the cyclical component and the noise component.

If, at the outset, a trend-cycle component has been extracted from the data via the Butterworth filter, then the application of the seasonal adjustment filter will result in a threefold decomposition of the data, which will include the trend-cycle, the seasonal component and the residual noise component.

1.5 **The Compound Filters**

Inspection the periodogram of the residual deviations of the data from a polynomial trend function or from a trend-cycle function extracted by the Butterworth filter may reveal that the basic filter will fail to remove some of the elements that are contributing to the seasonal fluctuations. In that case, a compound filter may be called for that is also targeted at frequencies adjacent to the seasonal frequencies.

The program provides a double compound filter. This comprises two filters applied in series, which are targeted at frequencies that are offset on either side of the seasonal frequencies. The offsets, which are limited to 4° degrees, are the same for both filters. An adverse effect of this design is that it allows minor leakages to occur at the seasonal frequencies, whereby elements that should be nullified are allowed to pass into the seasonally adjusted data. The problem becomes more severe as the offsets are increased.

A triple filter is also provided, which supplements the basic seasonal adjustment filter by two offsets filters, which are targeted at frequencies above and below the seasonal frequencies. In this case, the offsets on either side of each of the seasonal frequencies may be specified individually.

The polynomial operators of the basic filter, which correspond to the moving-average and feedback operations. have simple forms that are immediately accessible. By contrast, the polynomial operators of the offset filters are compounded from their quadratic factors, which incorporate the offset frequency values. The flexibility of the triple filter arises from the possibility of specifying each of these values individually.

1.6 **Operating the Program**

The program is operated by typing the numbers and letters that are written beside the menu options, by answering *Yes* or *No (Y/N)* to various questions and by entering the required parameters and numerical values. The procedures of the program are either self-explanatory or else they are explained in the appropriate places by short texts displayed on the computer screen. This should enable a user to operate the program without the aid of this manual.

For many of the procedures, pre-requisite information must be supplied or prior choices must be made. If any such items are missing, then the program will ask for them. Thus, for example, if the procedure to plot a graph is activated before any data have been provided and before the parameters of the graph have been declared, then the program will ask the user to provide these items.

The program is also protected again inappropriate inputs. Thus, if a non-numeric symbol is typed when a bounded integer has been called for, then the program will persist in asking for the integer until one that satisfies the bounds has been provided.

This manual lists the menus of the program, and it gives a thematic account of the facilities of the program. It should serve as a ready reference. The various procedures that are described in this account are accessed via the sub menus of the program. The pathways to these procedures are listed under their headlines in the following manner:

(SEADOS.PAS → Primary Menu → Secondary Menu →Procedure)

The program resides is a zip file at the following address:

http://www.sigmapi.org.uk/seados.zip/

The code of the program, which is in *Pascal,* can be found within the zip file, which also contains a collection of data and an additional copy of this manual.

2 THE MENUS OF THE PROGRAM

The following is a summary of the menus and sub-menus of the program. Most of the items of the main menu of *SEADOS.PAS* subsume a sub-menu. Secondary sub-menus are not listed:

SEADOS.PAS: Enhanced Methods of Seasonal Adjustment
1. Page Parameters
2. Get the Data, Transform the Data
3. Plot the Data
4. Detrend the Data
5. Specify the Seasonal Adjustment Filter
6. Do Seasonal Adjustment
7. Save and Exit

The sub menus associated with the primary menus are as follows:

1. PAGE PARAMETERS

2. GET THE DATA, TRANSFORM THE DATA
1. Read the Data
2. Take Logarithms of the Data
3. Extract a Polynomial Trend
4. Plot the Data
5. Plot the Periodograms of the Data and its Components
6. Return to the Main Menu

3. PLOT THE DATA
1. Plot the Data Sequence
2. Return to the Main Menu

4. DETREND THE DATA
1. Extract a Polynomial Trend
2. Specify the Butterworth Filter
3. Apply the Butterworth Filter
4. Return to the Main Menu

5. SPECIFY THE SEASONAL ADJUSTMENT FILTER
1. The Basic Seasonal Adjustment Filter
2. The Basic Seasonal Adjustment Filter with Smoothing
3.The Double Offset Filter
4.The Triple Filter with Offsets
5. Return to the Main Menu

6. DO SEASONAL ADJUSTMENT
  1. Specify the Seasonal Adjustment Filter
  2 .Apply the Seasonal Adjustment Filter
  3. Return to the Main Menu

7. SAVE AND EXIT
  1. Save the Data Components
  2. Return to the Main Menu
  3. Exit the Program

3 DATA HANDLING

3.1 Data Entry

(SEADOS.PAS → Get the Data, Transform the Data → Read the Data)

The program relies on data files in the .txt format in which the elements are written on successive lines.

There is an allowance for an accompanying integer index on the same line, which must precede the data value. The index must have a constant increment in passing from one line to the next. The program will check whether or not this is the case and, if it is not, then it will cease to read the data.

Seasonal and Quarterly Data

A further allowance is made for monthly or quarterly data with four or twelve elements per line, respectively, separated by spaces. In such cases, the program will seek conformation that the intention is to read across successive the rows of the data file and to join the elements in a single sequence.

The Headline of the Data File

The first four lines of the data file are permitted to be a description of the data in numerical and non-numerical text. If non-numerical text is found in subsequent lines, then the program will alert the user; and it will cease to read the data.

The Data Frequency

The user is asked to identify the frequency of the data by naming the interval between successive observations, which may be

(a) *Annual,*
(q) *Quarterly,*
(m) *Monthly,*
(n) *Other.*

The Name of the Data

The user may wish to identify the data by name. Otherwise, the data will be described as *an unidentified data series.* The naming of the data affects the legends that are displayed below the graphs that may be plotted on the screen. It also affects the tags that are applied to the *PostScript* code that can be generated while plotting graphs and diagrams on the screen.

3.2 **Data Plotting**

(SEADOS.PAS → Get the Data, Transform the Data → Plot the Data)

Before plotting the data, the user is asked to declare whether or not they wish to produce an accompanying *PostScript* file. If the answer is in the affirmative, then the choice can be made of producing either an *Encapsulated PostScript* code or a code in the *Textures format.*

Thereafter, it will be necessary to specify the page parameters of the graph, which is a matter of choosing the dimensions of the frame that surrounds the graph. See *Page Parameters* in section 4.

3.3 **Data Transformations**

(SEADOS.PAS → Get the Data, Transform the Data → Take Logarithms of the Data)

The program automatically determines whether the data have a significant trend and whether their mean value is significantly different from zero.

Extracting a Polynomial Trend

If there is a trend in the data, then it will be appropriate to interpolate a polynomial trend function by a least-squares regression and to create a sequence of residual deviations, which will have a mean of zero. More details are to be found in Section 6. If there is no trend in the data, then it may be appropriate to subtract the mean. This can be done by fitting a polynomial of degree zero.

Take Natural Logarithms

The program enables the user to take the natural logarithms (logarithms to the base e) of the data. If, at the outset, the logarithmic data sequence is required, then it will be unnecessary to ask to read the data before asking to take their logarithms. Asking for logarithms to be taken causes the data to be read, if they are not already available. The logarithmic transformation can be reversed in the process of saving the components of the decomposition of the data.

4 GRAPH PLOTTING

4.1 **Page Parameters**

(SEADOS.PAS → Page Parameters)

The user must first declare whether or not they require a *PostScript* code to be generated to accompany the screen graphics. If the answer is in the affirmative, then the choice can be made of producing an *Encapsulated Postscript* (.eps) code or a code in the *Textures* format. (The latter relates to an implementation of the TEX typesetting program that is no longer commercially available. It has been used in typesetting this manual.)

The graph is plotted in a frame of which the user must supply the dimensions. The available width of the frame is from a minimum of 5cm to a maximum of 13.5 cm and the available height is from 3 cm to 9 cm.

It is assumed that the graph will be placed on a printed page of A4 dimensions. In that case, the advice is given that, to print at most two diagrams per page, the maximum dimensions should be 9 cm width times 6 cm height. For three diagrams per page, the maximum dimensions should be 9 cm times 3.75 cm.

If a graph is to be plotted only on the screen, then the maximum available dimensions may be chosen. The maximum dimensions will be obtained by entering 99, or any other excessive number, in response to the request for the width and the height.

In the *Windows* version of the program, a plotted graph can be dismissed usually by typing a *RETURN*. However, if the graphics page has been deactivated by clicking the mouse when the cursor is elsewhere on the computer screen, then it will be necessary to dismiss the graph via the check box in the upper right corner of the page.

4.2 **Plot The Series**

(SEADOS.PAS → Get the Data, Transform the Data → Plot the Data)

The program will plot the primary data sequence (or its logatrithm) that is held in memory, or of any other sequence that has been derived in the process of trend estimation or filtering.

If the data have been de-trended by interpolating a polynomial function, and if a seasonal adjustment procedure has been implemented, then the following menu items will be available:

1. *Plot the Data Sequence*
2. *Plot the Polynomial Residuals*
3. *Plot the Seasonal Component*
4. *Plot the Seasonally Adjusted Data*

If a trend-cycle function has been extracted by the Butterworth filter, prior to the seasonal adjustment, then the following items will be available:

1. *Plot the Data Sequence*
2. *Plot the Trend-Cycle Function*
3. *Plot the Seasonal Component*
4. *Plot the Residue of the Threefold Decomposition*

5 DATA CHARACTERISTICS

5.1 The Periodograms

(SEADOS.PAS → Get the Data, Transform the Data → Plot the Periodogram of the Data and their Components)

The periodograms are an essential accompaniment of the processes of seasonal adjustment. Both the twofold and the threefold decompositions of the data rely on the periodogram of the residual deviations from an interpolated polynomial trend.

In the case of the twofold decomposition, the periodogram of the polynomial residuals is the means of assessing the frequency range of the elements that are contributing to the seasonal fluctuations. To assist in the specification of the seasonal adjustment filter, the periodiogram of the residuals provides a background to the plot of frequency response function the filter.

In the case of a threefold decomposition, the periodogram of the residuals will indicate the extent of the elements that are contributing to a low-frequency cycle within the data. The cycle can then be removed by the low pass Butterwoth filter, for which the cut-off frequency should be chosen to exceed the highest frequency within the cycle.

Thereafter, the specification of the seasonal adjustment filter can be guided by the periodgram of the residual deviations of the data from the trend-cycle function, which displays the frequency signature of the seasonal fluctuations as a background to the plot of frequency response function the filter.

A twofold decomposition of the data generates a seasonal component and a seasonally adjusted data sequence. When the decomposition has been accomplished, the following commands will be available:

1. *Plot the Periodogram of the Data*
2. *Plot the Periodogram of the Polynomial Residuals*
3. *Plot the Periodogram of the Seasonal Component*

A threefold decomposition of the data generates a seasonal component, a trend-cycle function and a residual noise component. When the decomposition has been accomplished, the following commands will be available:

1. *Plot the Periodogram of the Data*
2. *Plot the Periodogram of the Butterworth Residuals*
3. *Plot the Periodogram of the Seasonal Component*
4. *Plot the Periodogram of the Threefold Residue*

6 TRENDS AND TREND-CYCLE FUNCTIONS

6.1 Polynomial Regression

(SEADOS.PAS → DeTrend the Data → Polynomial Regression)

Ordinary Polynomial Regression

A numerically stable procedure is available for fitting a polynomial function to the data. The procedure avoids using the powers of the polynomial argument, which can give rise to numerical instability. Instead, it employs a sequence of orthogonal polynomials. Once the coefficients associated with the orthogonal polynomials have been determined, they are transformed into the power series coefficients.

Weighted Polynomial Regression

The purpose of a weighted regression is to ensure that the fitted polynomial will pass through the midst of the scatters of points at either end of the data sequence. To achieve this, the polynomial is fitted by a weighted least-squares regression, in which the maximum weight is given to the ends of the data sequence and the minimum weight is given to its mid point.

The procedure will ensure that there is no major disjunction in the circularised data sequence at the point where the head of the sample joins the tail. Such a disjunction can adversely affect the appraisal of the periodogram of the residual deviations of the data from the polynomial function.

A Variety of Weighting Functions

The user is first asked to specify the maximum and the minimum weights—the latter being unity by default. They are also asked to specify a U-shaped profile according to which the weights make their transition from the minimum to the maximum values.

Two such choices are a quadratic function and a crenellated function that jumps abruptly from the minimum value, which is maintained over an interval centred on the mid point, to the maximum value, which, by default, is the value in the first and the last quarters of the sample.

There is also an option for splitting the quadratic by interpolating a constant segment into an interval centred on the mid point. The equivalent option in respect of the crenelated function is to allow variations in the length of the segment over which the weighting function assumes its minimum value.

6.2 The Butterworth Filter and the Trend-Cycle Function

(SEADOS.PAS → Detrend the Data → Butterworth Filter)

The lowpass Butterworth filter, which operates directly on the data, will extract a combined trend-cycle function. It will generate a residual sequence that comprises the seasonal component and the noise component. It is specified by declaring the cut-off frequency, which marks the midpoint of the transition of the filter, and the filter order, which governs the rate of transition.

The Frequency Response of the Filter

The low pass Butterworth filter is capable of a rapid transition from a low-frequency pass band to a high-frequency stop band, thereby isolating a low-frequency spectral structure with a well-defined upper limit. The nominal cut-off point can be determined in view of the peridogram of the residual deviations of the data from a polynomial interpolated by least-squares regression. If the polynomial residuals are available, this periodogram will be plotted as a background to the frequency response function.

The Application of the Filter

When the filter is applied to a trended data sequence, the program will display a graph of the data with an interpolated smooth trajectory, described as the trend-cycle component. When this graph has been dismissed, the program will display a graph of the residual deviations of the data from the trend-cycle component.

7 SEASONAL ADJUSTMENT IN THE TIME DOMAIN

The program provides a variety of filters for the seasonal adjustment of the data that operate in the time domain, including a basic filter that is designed to imitate the affects of the conventional model-based filters that are exemplified, for example, by the *SEATS–TRAMO* program.

In a twofold decomposition of the data, filters operate on the residuals from a polynomial de-trending. They nullify the seasonal component within the residual sequences, leaving a vector of seasonally-adjusted residuals.

The sequence of seasonal fluctuations is the complement, within the sequence of polynomial residuals, of the sequence of seasonally-adjusted residuals. Subtracting it from the data sequence creates the seasonally-adjusted version. Equally, the seasonally adjusted residuals can be added to the detrending polynomial to create the seasonally-adjusted data.

In a threefold decomposition of the of data, the filters operate on the residual deviations from a trend-cycle function extracted by the Butterworth filter. The decomposition comprises the trend-cycle function, which stands for the seasonally adjusted data, the seasonal component and a residual noise component, which is the complement of the seasonal component within the sequence of trend-cycle residuals. These residuals contain the same seasonal component as do the residuals from a polynomial de-trending; and it will be extracted by the same seasonal adjustment filters.

The basic filter eliminates the elements at the seasonal frequencies and it attenuates the adjacent elements to an extent that decreases with the distance from the seasonal frequencies.

The enhanced double and triple time-domain filters are intended to suppress the elements adjacent to the seasonal frequencies more firmly than is possible using the basic filter. In the case of the double filter, this is achieved by eliminating elements at frequencies on either side of the seasonal frequencies. In the case of the triple filter, the seasonal frequencies are targeted together with frequencies on either side that are at distances that can be determined individually in every instance.

A trend-cycle function can also be created by applying a smoothing filter to the seasonally adjusted residuals created by the basic filter. Adding this seasonally adjusted and smoothed component back to the polynomial function creates a trend-cycle function. This represents an attempt to mimic one of the products of a conventional method of seasonal adjustment.

7.1 **The Basic Seasonal-Adjustment Filter**

(SEADOS.PAS → Do Seasonal Adjustment → The Basic Seasonal-Adjustment Filter)

A facility is provided for the seasonal adjustment of monthly or quarterly data showing a pattern of seasonal variation. A so-called comb filter is provided, which has a frequency response with notches that extend to zero at the seasonal frequencies. These notches serve to eliminate the elements of the data at the seasonal frequencies and to attenuate the elements at the adjacent frequencies.

The width of the notches are affected by a smoothing parameter $\lambda \in (0, 1)$, and they become narrower as $\lambda \to 0$. A default value of 0.5 has been given to this parameter.

A second parameter, described as the pole parameter $\rho \in (0, 1)$, must also be specified. This determines the modulus of the poles of the filter, which serve to counteract the effects of the zeros at frequencies that lie between the seasonal frequencies. The closer is ρ to unity, the narrower are the notches or clefts of the frequency response function of the filter. A default value of 0.9 may be been given to this parameter.

Before applying a seasonal adjustment filter, the user is constrained to remove any trend in the data by a polynomial regression or by the Butterworth lowpass filter.

De-trending the Data

The de-trended data are the residual deviations from a interpolated polynomial or from a trend-cycle function extracted by the Butterworth filter. If a polynomial has been fitted to the data, then the periodogram of the residual deviations can serve as a quide in determining a nominal cut-off point for the lowpass Butterworth filter. In that case, the periodogram will be plotted as a background onto which the response of the Butterworth filter will be superimposed.

The upper limit of the pass band of the Butterworth filter should fall short of the fundamental seasonal frequency, which is liable to be be marked in the periodogram by a tall spike. In order to accomodate the transition from the pass band to the stop band, the nominal cut-off of the filter should be less than this limiting value.

The Frequency Response of the Seasonal Adjustment Filter

The program displays the frequency response of the filter. If a polynomial trend function or a trend-cycle function has been extracted from the data, then the frequency response will be superimposed on the periodogram of the residual deviations of the data from that function.

The Application of the Filter

Applying the filter to residuals of a polynomial de-trending will lead to a twofold decomposition of the data. A plot of the polynomial residuals will be displayed with a seasonally adjusted version superimposed as a red line. Then a plot of the seasonal component that has been extracted from the polynomial residuals will be displayed. This is followed by a graph that shows the seasonally adjusted data, represented by a red line, superimposed on the data sequence. The seasonally adjusted data are created by adding the seasonally adjusted version of the residual sequence to the de-trendind polynomial.

Applying the Butterworth filter to the data to extract a trend-cycle function will initiate a threefold decomposition. The application of a seasonal adjustment filter to the trend-cycle residuals will generate a graph of those residuals with a superimposed seasonally adjusted sequence, which represents the residual noise component of the threefold decomposition. On closing the graph, the residual noise sequence is plotted on it own, whereafter a graph of the seasonal component will be presented.

7.2 **The Basic Seasonal-Adjustment Filter with Smoothing**

(SEADOS.PAS → Do Seasonal Adjustment → The Basic Filter with Smoothing)

A trend-cycle function, resembling a function generated by the conventional seasonal adjustment procedures, is created by applying a lowpass smoothing filter to the seasonally adjusted residuals, generated by the basic filter, before they are added back to the polynomial function that has served to de-trend the data. A bi-directional moving-average filter is employed in smoothing the seasonally-adjusted residuals.

The combination of the basic filter with the smoothing filter may be described as the tapered filter, in reference to the profile of its frequency response function.

Three parameters must be specified. In addition to the smoothing parameter and the pole parameter, which may be given default values of 0.5 and 0.8 respectively, the user must specify a value for the parameter μ of the smoothing filter. (A value of 0.7 is suggested.)

De-trending the Data

Before applying a seasonal adjustment filter, the user must remove any trend in the data by a polynomial regression. It is inappropriate, in this context, to de-trend the data by using Butterworth filter to interpolate a trend-cycle function, since it is purpose of the tapered filter is to generate a trend-cycle function.

Neverthless, the preferred method for estimating a trend-cycle function is via the Butterworth filter. This will lead to a threefold decomposition of the data when a seasonal adjustment filter is applied to the residual deviations of the data from the Butterworth trend-cycle function,

The Frequency Response of the Tapered Filter

The frequency response of the smoothing filter is displayed. This is compounded with the frequency response of the ordinary seasonal-adjustment procedure to produce the frequency response of the tapered filter, to be used in estimating the trend-cycle function. If a polynomial trend function has been extracted from the data already, then the frequency response of the filter will be superimposed on the periodogram of the residual deviations of the data from the polynomial.

The Application of the Filter

On applying the filter to the de-trended data, the program will display the smoothed cyclical component, in red, superimposed on the sequence of residuals from the polynomial detrending. Thereafter, a residual sequence, obtained by subtracting the cyclical from the polynomial residuals, is displayed. The trend-cycle function is created adding this residual sequence to the polynomial function that has served to de-trend the data.

7.3 **The Double Offset Seasonal-Adjustment Filter**

(SEADOS.PAS → Do Seasonal Adjustment → The Double Offset Filter)

The double offset seasonal-adjustment filter comprises two filters that are applied in sequence. The filter has poles and zeros that are at frequencies at small distances below and above the seasonal frequencies.

The intention is to widen the stop bands that surround the seasonal frequencies. An adverse effect of this design is to allow leakage to occur at the seasonal frequencies, which will become more severe as the offsets are increased.

Three parameters are required. In addition to the smoothing parameter and the pole parameter, which may be given default values of 0.5 and 0.8 respectively, the user must specify the offset of the filters as a number of degrees.

The same offset is applied to both filters in respect of all of the seasonal frequencies. The first filter is offset below the seasonal frequencies and the second filter is offset above the seasonal frequencies. The maximum allowable offset is 4° degrees.

De-trending the Data

The data may be de-trended either by interpolating a polynomial function or by using the Butterworth filter to extract a trend-cycle function. The residual deviations of the data from the polynomial or from the trend-cycle function constitute the de-trended data.

The Frequency Response of the Double Filter

The program displays the frequency response of the filter superimposed on the periodogram of the residual deviations—i.e. the detrended data— whenever this is available. The leakage of the filter at the seasonal frequencies may be noted. If a polynomial trend function or a trend-cycle function has been extracted from the data, then the frequency response will be superimposed on the periodogram of the residual deviations of the data from that function.

The Application of the Filter

If the data have be de-trended by a polynomial interpolation, then, on applying the filter, the program will display the residual sequence with the seasonal adjusted version superimposed as a red line. Thereafter, the seasonal component extracted from the residual sequence will be displayed. Then, the seasonally adjusted data sequence will be displayed, which is created by adding the seasonally adjusted residual sequence to the polynomial function that has been interpolated through the data.

If a trend cycle function has been extracted from the data, then this function will stand in place of the seasonally adjusted data. Applying the filter to the residual deviations from the trend-cycle function will create a seasonally adjusted version of the deviations, which represent the noise component of a threefold decomposition. This is plotted, at first, as a red line superimposed on the residual deviations. Thereafter, it is potted on its own. This will be followed by a graph of the seasonal component.

7.4 **The Triple Seasonal-Adjustment Filter with Offsets**

(SEADOS.PAS → Specify the Seasonal Adjustment Filter → Do Seasonal Adjustment → The Triple Filter with Offsets)

The triple filter supplements the ordinary seasonal adjustment filter by two offset filters with their pole-zero pairs below and above the seasonal frequencies. The offsets below and above each of the seasonal frequencies may be specified individually in numbers of degrees.

The values of the offsets are to be determined by the user in view of the periodogram of the de-trended data, which are the residuals from a polynomial regression or from a trend-cycle function estimated by the Butterworth Filter. The objective is to create stop bands and clefts surrounding the seasonal frequencies that are wide enough to encompass all of the elements that contribute to the seasonal fluctuations.

In addition to the values of the offsets, the user must specify the smoothing parameter and the pole parameter, which are given default values of 0.5 and 0.8.

De-trending the Data

The de-trended data are the residual deviations from a interpolated polynomial or from a trend-cycle function extracted by the Butterworth filter. If a polynomial has been fitted to the data then the periodogram of the residual deviations can serve as a quide in determining a nominal cut-off point for the lowpass Butterworth filter. In that case, the periodogram will be plotted as a background onto which the response of the Butterworth filter will be superimposed.

The Frequency Response of the Seasonal Adjustment Filter

The program displays the frequency response of the filter. If a polynomial trend function or a trend-cycle function has been extracted from the data, then the frequency response will be superimposed on the periodogram of the residual deviations of the data from that function. The combined graph should indicate whether the clefts in the frequency response will be sufficient to eliminate the seasonal fluctuations from the data.

The Application of the Filter

After the frequency response of the filter has been displayed, the filter can be applied to the de-trended data. If the data have not been de-trended, then the program will demand that polynomial function should be fitted to the data

The program will proceed to display the sequence of residual deviations from the polynomial trend or from the trend-cycle function, with a seasonally adjusted version superimposed as a red line. Next the seasonal component that has been extracted from the residuals will be displayed

If data have been de-trended by interpolating a polynomial function, then the seasonally adjusted data sequence will be displayed, which is created by adding the seasonally adjusted residuals of the poynomial de-trending to the polynomial function,

If the Butterworth filter has been used in extracting trend-cycle function from the data, then the residual sequence from de-trending via the Butterworth filter will be displayed with a superimposed seasonally adjusted version, which is the residual sequence of a threefold decomposition of the data. Then, this residual sequence will be displayed on its own, followed by the seasonal component extracted from the trend-cycle residuals.

8 CASE STUDY 1: The Basic Filter

In this case study, the basic filter is applied to a tractable data sequence. The result, in the first instance, is a twofold decomposition of the data that produces a regular pattern of seasonal fluctuations and a sequence of seasonally adjusted data with a rough profile. Thereafter, a threefold decomposition of the data is pursued, which creates a smooth trend-cycle function and a residual noise sequence, in addition to the regular seasonal component

The data sequence in question, within *PLANES.txt*, consists 241 monthly records of the numbers boarding U.S international air flights. These are displayed in Figure 1.

8.1 Reading and Displaying the Data

Before displaying the data, the page parameters must be supplied via the item *1. Page Parameters* on the principal page of the program that bears the legend *SEADOS.PAS: Enhanced Seasonal Adjustment.*

The question *Do you want to output in Postscript?* is posed, to which an answer *Y* or *N* must be supplied. Thereafter, the user is required to specify the height and the width in centimetres of the box that bounds the graph. The answers should be determined by whether or not the *PostScript* code for a printed graph is required.

If the answer is no (*N*), then it is appropriate to produce a screen image of the maximum dimensions. This can be achieved by supplying values that

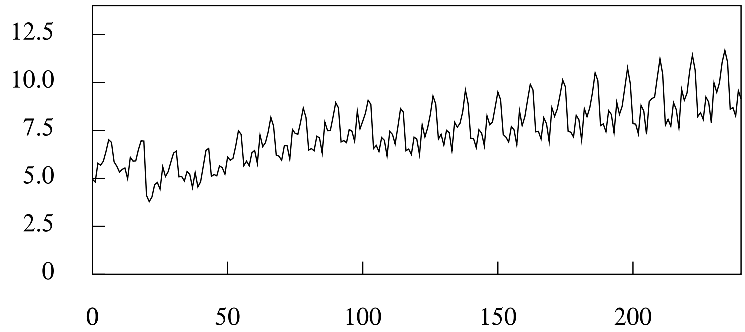

Fig. 1 241 monthly records of the numbers of passengers, in thousands, boarding U.S international air flights, between January 2000 and January 2020 inclusive.

exceed the maximum dimensions. It is appropriate, in that case, to type 99 for both the width and the height.

Next, the data can be read via the commands *2. Get the Data, Transform the Data* $\longrightarrow$ *1. Read the Data.* The name of the file to be supplied to the program, which is *PLANES.txt*, can be typed without regard to the capitalization of the letters. An alert will be posted on account of the presence of the textual description at the top of the file. This can be ignored and, to proceed, a $\langle RETURN \rangle$ can be typed.

The program will also alert the user that, in this case, the data are preceded by an index, which will be ignored. Since the intervals between observations are monthly, the letter M (or m) should be typed in response to the question *What is the interval between observations?* In response to the question of whether the data should be given a name other than *an unidentified data sequence,* the answer N can be typed.

Now, the data can be displayed via the commanda *4. Plot the Data* $\longrightarrow$ *1. Plot the Data Sequence.* The graph can be dismissed either by typing a $\langle RETURN \rangle$ or by checking the close box in the upper right-hand corner, in case the graphics screen has been partially de-activated by clicking elsewhere on the computer screen. Thereafter, the command *2. Return to the Main Menu* will take the program back to the *GET THE DATA* menu.

8.2 **Transforming and De-trending the Data**

In view of the trended nature of the data and of the increasing amplitude of its fluctuations, it is appropriate to take logarithms and, thereafter, to interpolate a polynomial of degree 4. This can be achieved via *GET THE DATA* menu, where the items *2. Take Logarithms of the Data* and *3. Extract a Polynomial Trend* can be accessed in succession. The latter will give rise to a graph of the logarithmic sequence with the interpolated polynomial superimposed. This is

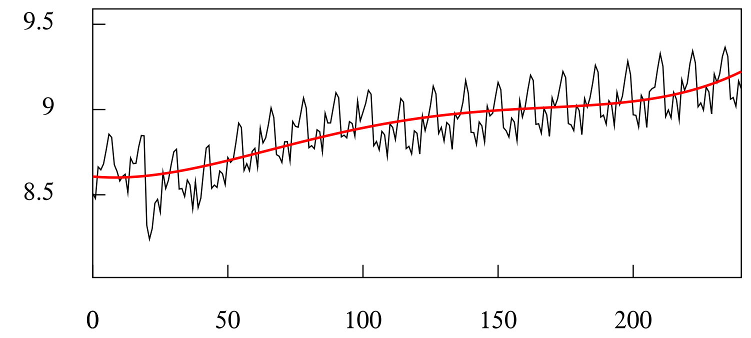

Fig. 2 The logarithms of the numbers of passengers boarding U.S international air flights, with a superimposed polynomial trend function of degree four.

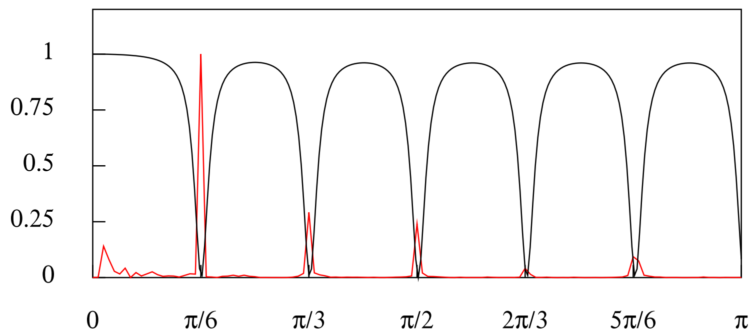

Fig. 3 The frequency-response functions of the ordinary seasonal-adjustment filter for monthly data with $\lambda = 0.5$. and $\rho = 0.9$, superimposed on the periodogram of the residuals from the polynomial de-trending of the logarithmic airline passenger data.

shown in Figure 2. On dismissing this graph, a graph of the residual sequence will be displayed.

The combined effect of the two data transformations can be assessed via *5. Plot the Periodogram of the Data and its Components* $\longrightarrow$ *2. Plot the Periodogram of the Polynomial Residuals.* The resulting periodogram of Figure 3 shows a succession of spikes located at the seasonal frequencies. These are preceded by the spectral signature of some low-frequency fluctuations.

At this stage, there are alternative paths that can be taken, which will be followed in succession in this exercise. First, one must to return the main *SEADOS.PAS* menu.

8.3 **The Twofold Decomposition of the Data**

The program allows the specification of a seasonal adjustment filter to be explored before it is applied to the data. The relevant command is *5. Specify*

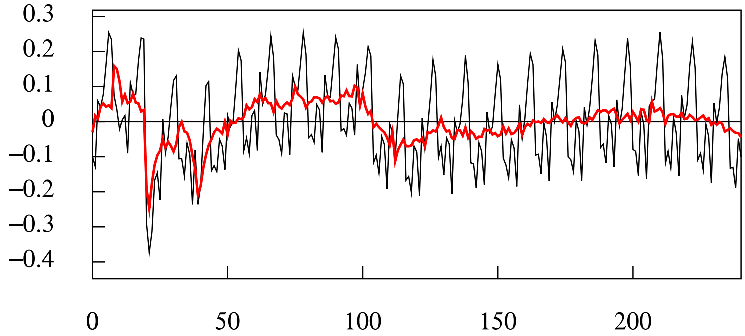

Fig. 4 The residual sequence from the polynomial de-trending of the logarithmic passenger data, with a superimposed seasonally adjusted sequence.

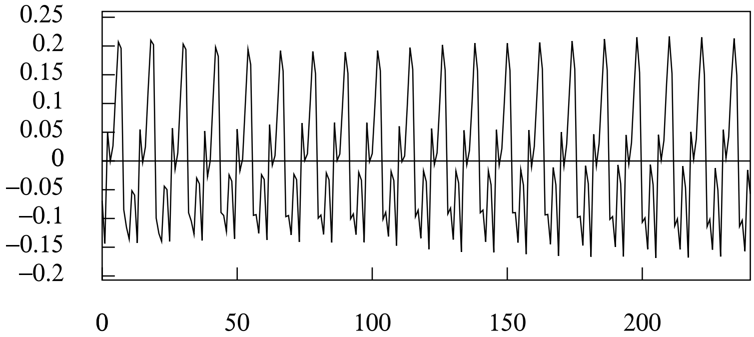

Fig. 5 The seasonal component extracted from the residual sequence from the polynomial de-trending of the logarithmic airline passenger data.

the Seasonal Adjustment Filter, albeit that, if the specification has not been provided in advance of the the command *6. Do Seasonal Adjustment,* it will be demanded as a prerequisite.

First, one should agree to equate the seasonal-adjustment frequency to that of the data. Then, one should opt for the item *1. The Basic Seasonal Adjustment Filter.* The filter has two parameters. The values suggested by the program may be accepted, which are $lambda = 0.5$ and $rho = 0.9$.

The result is a graph bearing the legend

The frequency response of a time-domain filter for the seasonal adjustment of a monthly data sequence, superimposed on the periodogram of the polynomial residuals.

This is Figure 3. On dismissing the graph, the program returns to the principal page, from which the command *6. Do Seasonal Adjustment* can be activated, followed by the command *2. Apply the Seasonal Adjustment Filter.*

The effect of the filter is to produce a seasonally adjusted version of the sequence of polynomial residuals, which is superimposed as a red line on the

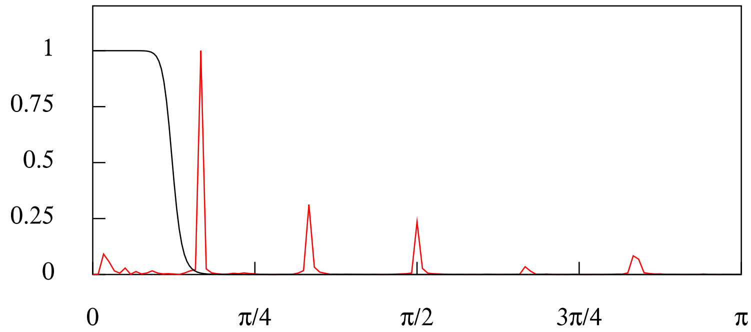

Fig. 6 The frequency response of a Butterworth filter of order 8, with a nominal cut-off frequency of $\pi/8$ or 22.5°, superimposed on the periodogram of the residual deviations from a polynomial interpolation of the logarithmic airline passenger data.

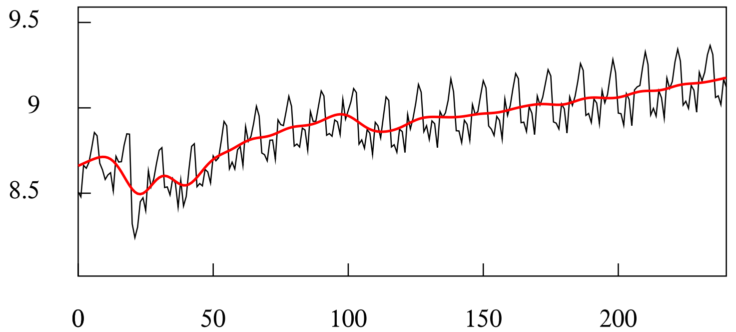

Fig. 7 The smooth trend cycle function extracted from the logarithmic airline passenger data by a Butterworth filter of order 8 and with a nominal cut-off frequency of $\pi/8$ or 22.5°.

residuals. This is shown in Figure 4. Thereafter, the seasonal component that has been extracted from the residuals is displayed, which is Figure 5. Next, the seasonally adjusted residuals are added back to the de-trending polynomial function to create a seasonally adjusted data sequence, which is superimposed on the plot of the data.

8.4 **The Threefold Decomposition of the Data**

On returning to the main menu, the second of the alternative decompositions can be pursued. However, before taking this path, one might look again at the periodogram of the residual sequence via the commands *2. Get the Data, Transform the Data* ⟶ *Plot the Periodogrm of the Data and the Components.*

The periodogram reveals that the spectral signature of the low frequency fluctuations is bounded by the fundamental seasonal frequency of $\pi/6 = 30°$. The commands *4. Detrend the Data* ⟶ *2. Specify the Butterworth Filter*

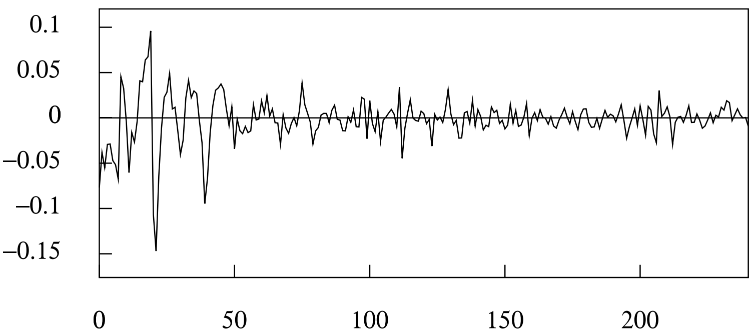

Fig. 8 The residual noise sequence of threefold decomposition of the logarithmic airline passenger data.

can now be issued from the main menu. The program allows the effects of alternative specifications of the filter to be investigated prior to its application. The nominal cutoff point of the low pass filter may be specified as 22.5° degrees ($\pi/8$ radians), and the order of the unidirectional filter may be specified as 8.

The frequency response of the Butterworth filter, which is plotted on the screen, is superimposed on the periodogram of the polynomial residuals. This is shown Figure 6. The trend-cycle function will be synthesised from the elements that are isolated by the pass band of the filter.

From the menu *DETREND THE DATA,* the command *3. Apply the Butterworth Filter* can be chosen. The immediate result is a graph of the trend-cycle function, in red, superimposed on the data. This is shown in Figure 7. Next, the sequence the deviations of the data from this function are displayed,

The alternative threefold decomposition can be pursued on returning to the main menus. The command *6. Do Seasonal Adjustment* leads to the *DO SEASONAL ADJUSTMENT* menu, from which *2. Apply the Seasonal Adjustment Filter* can be selected. The graph of the frequency response of the seasonal adjustment filter is displayed superimposed on the periodogram of the Butterworth trend-cycle residuals. It will be observed that the latter is devoid of the spectral signature of the low-frequency fluctuations.

Once the graph has been dismissed, the item *2. Apply the Seasonal Adjustment Filter* may be applied. A sequence of graphs ensues. The first of these shows the residual noise sequence of threefold decomposition superimposed on the Butterworth residuals. This is followed by a graph of the noise sequence on its own, which is Figure 7.

Next is the graph of the seasonal component that has been extracted from the Butterworth residuals. This will be found to be identical to Figure 5, which represents same the component, albeit that it was extracted previously from the residuals of a polynomial de-trending. The trend-cycle function, which has been plotted already, can be viewed again via *3. Plot the Data* $\longrightarrow$ *2. Plot the Trend-Cycle Function,* starting from the main menu.

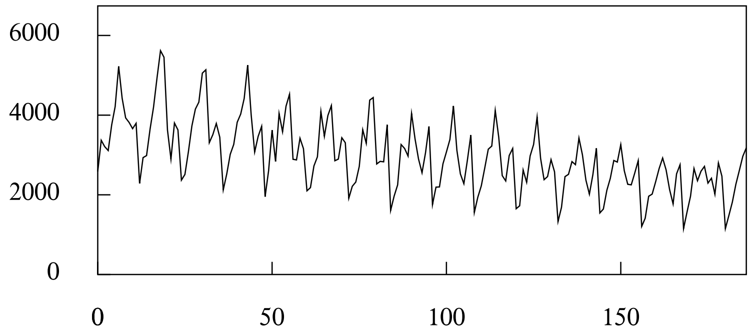

Fig. 9 187 values relating to the monthly sales in Australia of fortified wines from January 1980 to July 1995, measured in thousands of litres.

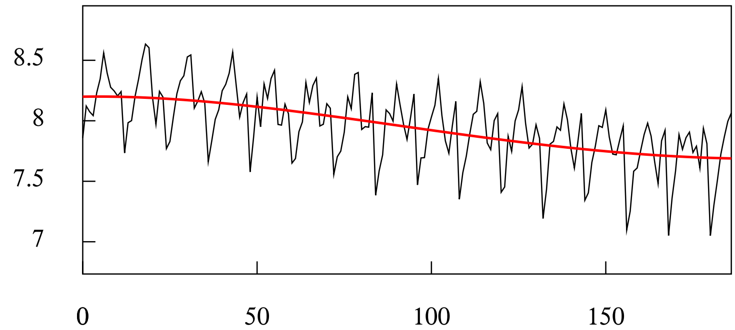

Fig. 10 The logarithms of the sales of fortified wines, with a superimposed polynomial trend function of degree four.

9 CASE STUDY 2: The Triple Filter

In the second case study, the triple filter is employed to eliminate elements adjacent to the seasonal frequencies.

9.1 **Transforming and De-trending the Data**

The data within *FORTIFY.txt* are 187 values relating to the monthly sales in Australia of fortified wines from January 1980 to July 1995, measured in thousands of litres. These volumes and their volatilities are decreasing over time; and it is appropriate to take their logarithms. The logarithmic transformation may be reversed at the conclusion of the analysis in the process of saving the components of the decomposition,

Figure 9 shows the original data and Figure 10 shows the logarithms of the data with an interpolated polynomial of degree 4. This function may be taken to represent the trend component. Extracting the trend component via

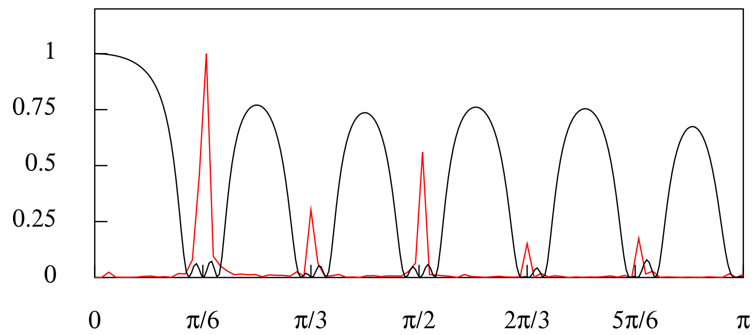

Fig. 11 The frequency-response functions of the triple seasonal-adjustment filter for monthly data with $\lambda = 0.5.$ and $\rho = 0.8$, superimposed on the periodogram of the residuals from the polynomial de-trending of the the logarithmic fortified wines data.

the Butterworth filter will produce a trajectory that barely departs from the polynomial trend.

9.2 **The Offset Filters**

Figure 11 shows the periodogram of the polynomial residuals with the frequency response of the triple seasonal adjustment filter superimposed. Here, the offset filters are targeted at frequencies at various distances from the seasonal frequencies. The offset frequencies below and above the seasonal frequencies are listed below:

Frequencies	Offset Below	Offset Above
$\pi/6 = 30°$	$4°$	$4°$
$\pi/3 = 60°$	$3°$	$4°$
$\pi/2 = 90°$	$4°$	$4°$
$2\pi/3 = 120°$	$2°$	$4°$
$5\pi/6 = 150°$	$2°$	$5°$
$\pi = 180°$	$3°$	–

The seasonal adjusted data that have been created by the basic filter are shown in Figure 12, whereas those created by the triple filter are shown in Figure 132, and the corresponding seasonal component is shown in Figure 14.

Compared with the basic filter, the triple filter has transferred into the seasonal component some of the noise that would otherwise be affecting the seasonally adjusted data. However, it is unclear that either of these versions of the seasonally adusted data are to be preferred to the polynomial trend function of Figure 10.

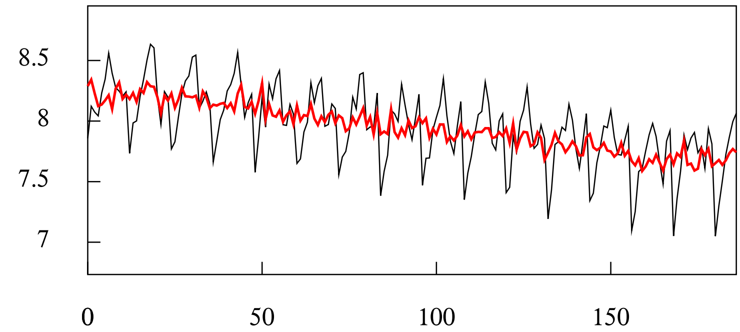

Fig. 12 The seasonally adjusted logarithmic fortified wine data created by the basic filter with $\lambda = 0.5$. and $\rho = 0.9$.

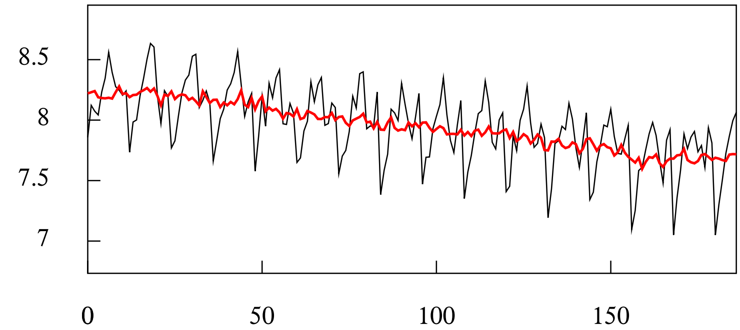

Fig. 13 The seasonally adjusted logarithmic fortified wine data created by the triple filter with $\lambda = 0.5$. and $\rho = 0.8$ with various offsets.

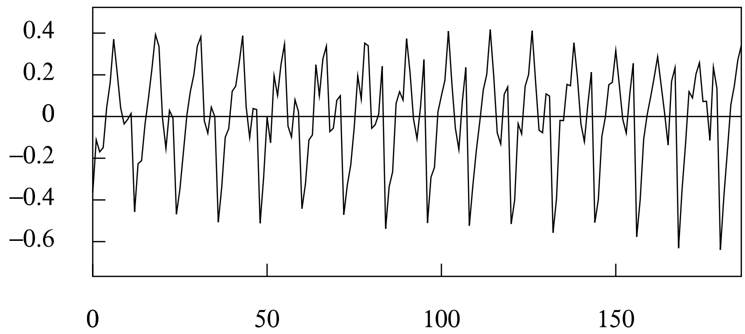

Fig. 14 The seasonally component extracted from the logarithmic fortified wine data by the triple filter.

10 THE CODE OF THE PROGRAM

The code of the program has been written in *Pascal* and it has been compiled with the *Free Pascal* compiler.

Free Pascal is an open source *Pascal* compiler, which can target a variety of processor architectures. The supported operating systems include *Windows* (16/32/64 bit, CE, and native NT), *Linux, Mac OS X, DOS* (16 bit, or 32 bit), *OS/2,* and others. Additionally, the *Free Pascal* team maintains a translater from *Pascal* to *Javascript* called *pas2js.*

The present version of the program operates on *Windows* machines under the *32 Bit DOS Protected Mode Interface.* It can be re-complied to operate under any of the alternative operating systems listed above.

The *Pascal* code accompanies the program in a *zip* file that also contains a manual and a collection of data. The units of the program bear the following names:

1. GLOBALS.PAS
2. MATHS.PAS
3. UTILS.PAS
4. LEGENDS.PAS
5. SCREEN.PAS
6. POST.PAS
7. DATASEG.PAS
8. BUTTER.PAS
9. FILTER.PAS
10. RESPONSE.PAS
11. ORGANISE.PAS
12. SEADOS.PAS

The ordering of these units in the list indicates their dependencies. Units lower in the list depend on those above them. Assuming that the units are located in a directory name `codefile` located on the `C` drive of the computer, the program can be compiled with the instruction

```
C:\codefile>fpc seados
```